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On account of the financial outlook a special letter was sent to the members of the Association in October, announcing the new membership dues and the need for subsidiary funds. It is our hope that the appeal for special contributions for 1921 expenses will not be forgotten as members pay their dues, for it is chiefly on this that we rely to offset the decrease in the reserve fund of the Association. It is gratifying to note here that a generous gift of $45.50 from President Smith and of $10.00 from Teachers College covers the expense of an effective campaign for members in America and in Europe, an expense that would otherwise fall directly on the Association treasury. It should also be noted that of 400 members who had paid their dues for 1921 before January first, sixty have made special contributions ranging from one up to seventeen dollars, a total thus far of $150.

While doubtless some may feel that they cannot do more than to pay the increased dues, it is within the range of possibility for almost any member to secure at least one new member; this in itself is a very helpful contribution to the finances of the Association and a real service to those not yet affiliated with the Association.

W. D. Cairns, *Secretary-Treasurer.*

ACOUSTIC CIRCLES.

By H. M. DADOURIAN, Trinity College.

The determination of the position of enemy artillery by *sound ranging* is one of the most interesting examples of the application of science to modern warfare. This consists in observing the time of arrival of the report, or the muzzle wave, of the enemy gun at a number of observation posts of known positions and then in using the *phonotelemetric* data thus obtained to determine the position of the gun. One of the simplest methods used in sound ranging is the following, known as the *method of concentric circles.*

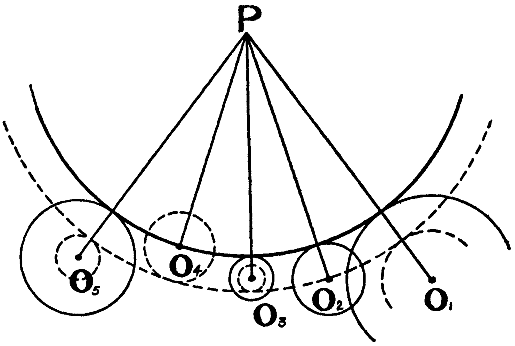

Fig. 1.

Let P, Fig. 1, denote the position of the enemy gun; O_1, O_2, etc., the positions of the observation posts at which the apparatus used for registering the arrival of the sound wave is placed; and T_1, T_2, etc., the times at which the sound wave reaches the posts O_1, O_2, etc., respectively. Furthermore suppose P, O_1, O_2, etc. to be in the same plane, with no intervening objects to obstruct the free propagation of the wave with uniform and constant velocity. Then it is evident that when a spherical wave originating at P reaches the nearest post O_i (O_4 in the case represented by Fig. 1) its distances from O_1, O_2, etc. will be $V(T_1 - T_i)$, $V(T_2 - T_i)$, etc., respectively, where V denotes the velocity of propagation of the wave. Therefore the circle about P passing through O_i is tangent to circles drawn about O_1, O_2, etc., having radii

$$\begin{aligned} R_{1i} &= V(T_1 - T_i), \\ R_{2i} &= V(T_2 - T_i), \\ &\cdots\cdots, \end{aligned} \tag{1}$$

respectively. This leads immediately to the method of concentric circles. On a drawing board points are located representing accurately the relative positions of the observation posts on a given scale, say 1 in 20,000; also a transparent sheet with closely drawn concentric circles is provided. When T_1, T_2, etc., are obtained circles are drawn about O_1, O_2, etc., with radii given by equations (1), the sheet with concentric circles is passed over the table until one of its circles becomes equally near tangency to all the circles about the points O_1, O_2, etc. Then the center of the concentric circles represents the position of the origin of the sound wave.

If in equations (1) O_i is not the post nearest P, then the right hand members of some of these equations become negative and the circle about P becomes tangent externally or internally to the circle about any post O_j according as $(T_j - T_i)$ is positive or negative, respectively. If in Fig. 1 O_2 is taken as the reference post, the circle passing through O_2 and having its center at P is tangent internally to the circles about O_3 and O_4, while it is tangent externally to the circles about O_1 and O_5 as is shown by the broken curves. Evidently the circle about the reference point has zero radius, consequently the tangency may be considered either as internal or as external or both.

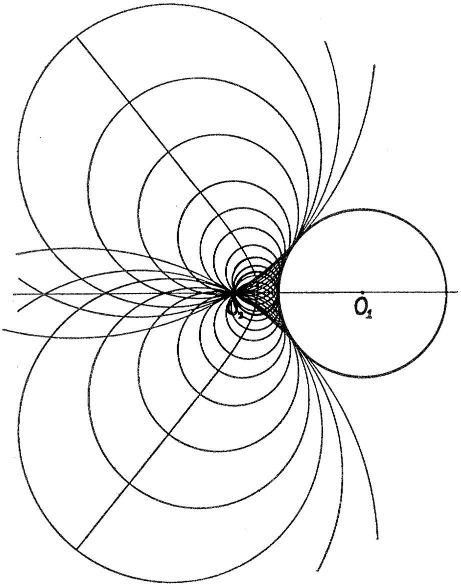

FIG. 2.

It is the object of this paper to show that the circle about P is common to a number of systems of circles which are determined by a set of phonotelemetric data. Consider two observation posts O_1 and O_2, Fig. 2, at which the arrival of a spherical sound wave is registered at the instants T_1 and T_2, respectively. Let r_1 and r_2 be, respectively, the distances of O_1 and O_2 from the origin of the wave; and t_1 and t_2 the intervals of time taken by the wave to reach the points O_1 and O_2, then

$$\begin{aligned} r_1 &= Vt_1, \\ r_2 &= Vt_2, \end{aligned}$$

and

$$r_1 - r_2 = V(t_1 - t_2).$$

But

$$t_1 - t_2 = T_1 - T_2.$$

Therefore

$$r_1 - r_2 = V(T_1 - T_2). \qquad (2)$$

Since the right-hand member is constant, equation (2) is the well-known equation of a hyperbola of which

$$R_{12} = V(T_1 - T_2)$$

is the transverse axis, and O_1 and O_2 are the foci. Thus the origin of the wave may be anywhere on the hyperbola of which (2) is the equation. Therefore representing, by means of a circle, the wave which starts from any point of the hyperbola and which reaches the posts O_1 and O_2 at T_1 and T_2, respectively, we obtain a system of circles which pass through O_2, are tangent to the circle of radius R_{12} drawn about O_1 as center, and have their centers on the hyperbola of which (2) is the equation. Equation (2) may also be regarded as the *acoustic equation* of the system of circles, and the system of circles may be called *acoustic circles*. The origin of the wave then is at the center of one of the circles of the system of acoustic circles of which (2) is the acoustic equation. If $T_1 > T_2$, the circles of the system have their centers on the left-hand branch of the hyperbola as in Fig. 2 and are tangent to the circle around O_1 externally. If on the other hand $T_1 < T_2$, the circles have their centers on the right-hand branch and are tangent to the circle around O_1 internally, as in Fig. 3. If the position of the source of sound is not limited to a plane, the system of acoustic circles becomes a system of spheres which have their centers on the hyperboloid of revolution obtained by revolving the hyperbola about the line O_1O_2 and which are tangent to the sphere of radius R_{12} drawn about O_1 as center.

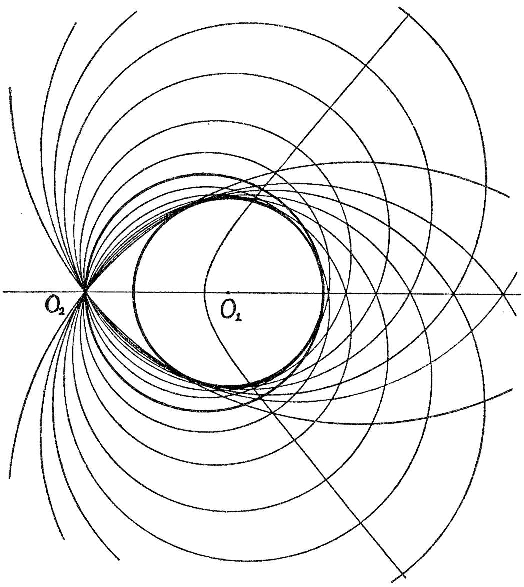

FIG. 3.

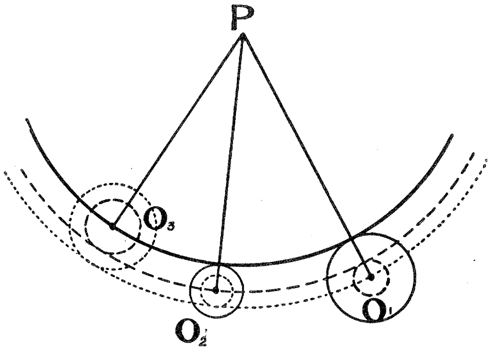

FIG. 4.

Now let T_3 be the time at which the wave front reaches another observation post O_3, then considering the data of two of the three posts at a time we obtain the equations

$$r_1 - r_2 = V(T_1 - T_2),$$

$$r_1 - r_3 = V(T_1 - T_3), \tag{3}$$

and

$$r_2 - r_3 = V(T_2 - T_3),$$

which are the acoustic equations of three systems of circles and of as many hyperbolas. The origin of the wave is at the common center of three circles, Fig. 4, one from each system. Each of the three circles passes through one of the points O, is tangent to circles about the other two and has its center at the common point of intersection of the hyperbolas given by equations (3).

In general, if the times of arrival of a wave at n posts of known positions are given, $n(n-1)/2$ equations can be obtained from the data, each of which forms the equation of a system of acoustic circles. (The truth of this statement may be easily seen by considering the number of permutations and combinations of n things taken two at a time). Evidently there are two circles in each system whose centers are at the origin of the wave. Of the $n(n-1)$ circles having their centers at the origin of the wave $(n-1)$ pass through O_1 and consequently are coincident; $(n-1)$ pass through O_2 and are coincident; and so on to the $(n-1)$ which pass through O_n. Therefore in general there are n distinct circles in the $n(n-1)/2$ systems which have their centers at the origin of the wave. The circle which passes through any post O_i is tangent to a circle of radius $V(T_j - T_i)$ drawn about the post O_j as center. The tangency is external if $T_j > T_i$ and internal if $T_j < T_i$.

ON THE NUMERICAL VALUE OF i^i.

By H. S. UHLER, Yale University.

The primary object of the present note is to place on record the values of $e^{-\pi/2}$ and of seven related powers of the same base which I have recently calculated to more than fifty decimal places.

In order to avoid the use of any table of mathematical constants the following series was employed

$$y \equiv e^{\sin^{-1} x} = 1 + x + \frac{x^2}{2!} + \frac{2x^3}{3!} + \frac{5x^4}{4!} + \cdots \; [x^2 < 1]$$

$$= (1 + \sum_1^\infty t_{2k+1}) + (x + \sum_1^\infty t_{2k+2}).$$

Since the collections of series at my disposal did not contain a formula for the general term of the above series it was necessary to prove that

$$t_{2k+1} = (0^2+1)(2^2+1)(4^2+1)\cdots[(2k-2)^2+1]x^{2k}/(2k)!$$